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# #WebinarsAEMAC



**22 de Febrero, 16 h (Madrid, GMT+2)**

WebinarsAEMAC Nº63

**Simulation and design of composite structures across the scales using physical and data-driven models.**


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Universidade do Porto


AEMAC

ASOCIACIÓN ESPAÑOLA DE MATERIALES COMPUESTOS

# *Simulation and design of composite materials and structures across the scales using physical and data-driven models*

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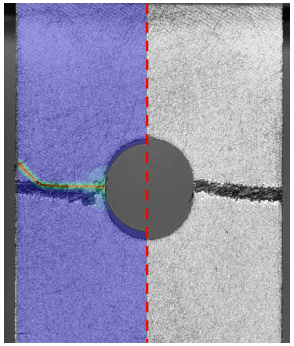
# Contents

*Important innovation drivers in the aerospace industry*

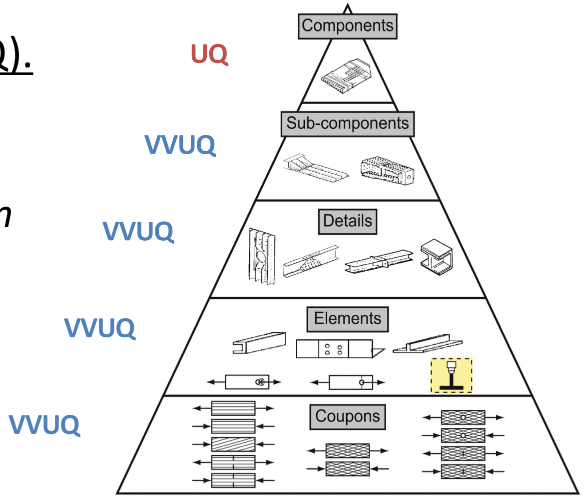
- Digitalization of processes/digital transformation, leading to reductions in cost and lead time. 50% reduction of certification costs by 2050 (ACARE vision for 2050). End-to-end simulation, including design, manufacturing and certification.



- Model verification, validation and uncertainty quantification (VVUQ).

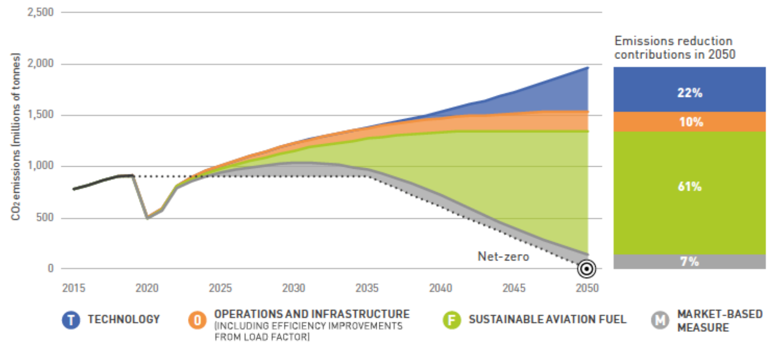
*"VVUQ are the primary methods for quantifying and building credibility in computational models".*

(J. Fatemi, Airbus NL, Porto, 2023)

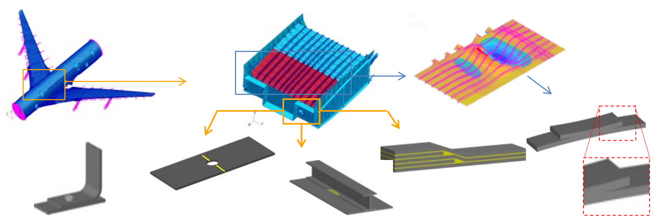


## *Important innovation drivers in the aerospace industry*

- Highly optimized composite structures & non-conventional composite materials, requiring simulation across the scales.

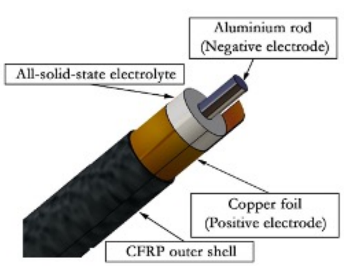


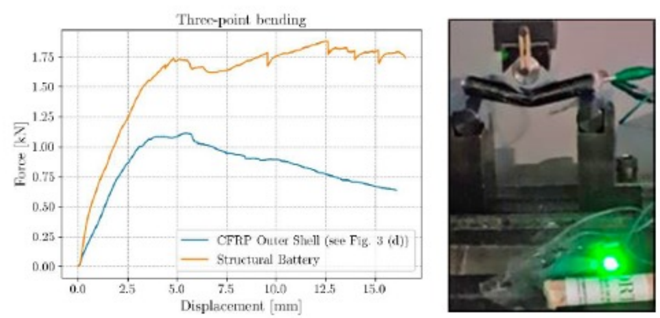

(Waypoint 2050, Air Transport Action Group, 2nd ed., Sept. 2021).



(Picture courtesy of Dr. Stéphane Mahdi, Airbus).

- Multifunctional composites (electrification, SHM).



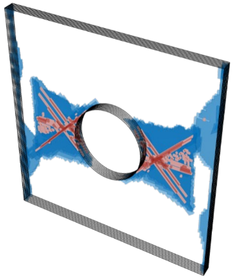




(a) (b)

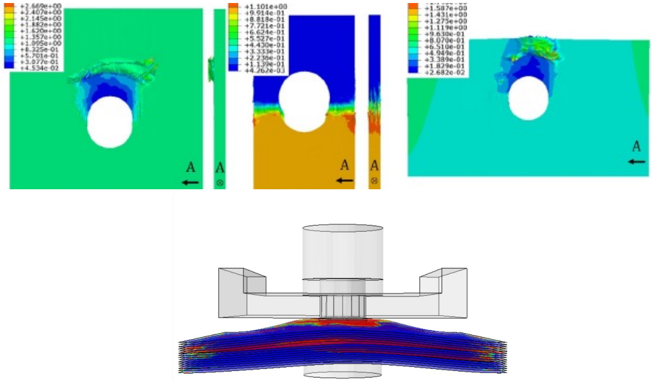
(F. Danzi, H. Braga, P.P. Camanho, Coaxial energy harvesting and storage, patent EP4324008)

# *Finite Element models*

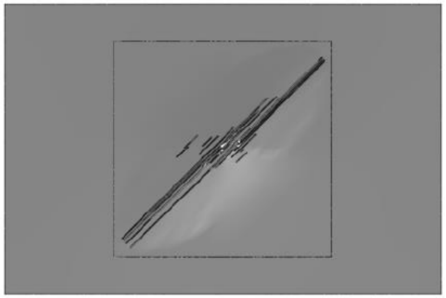
Open-hole tension/compression



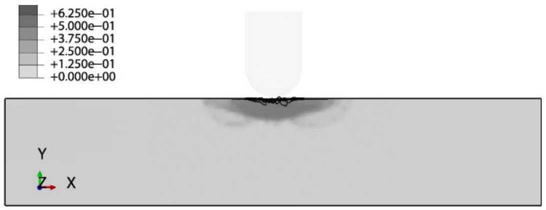
Bolted joint (in-plane and pull-through)



Low-velocity impact and CAI

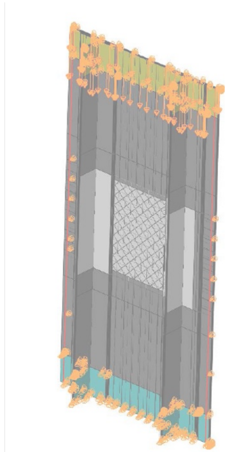


Low-velocity edge impact and CAEI




Lightning strike


CAI of stiffened panels



Computing times (16 CPUs) - between 1h and 80h; expert users required.

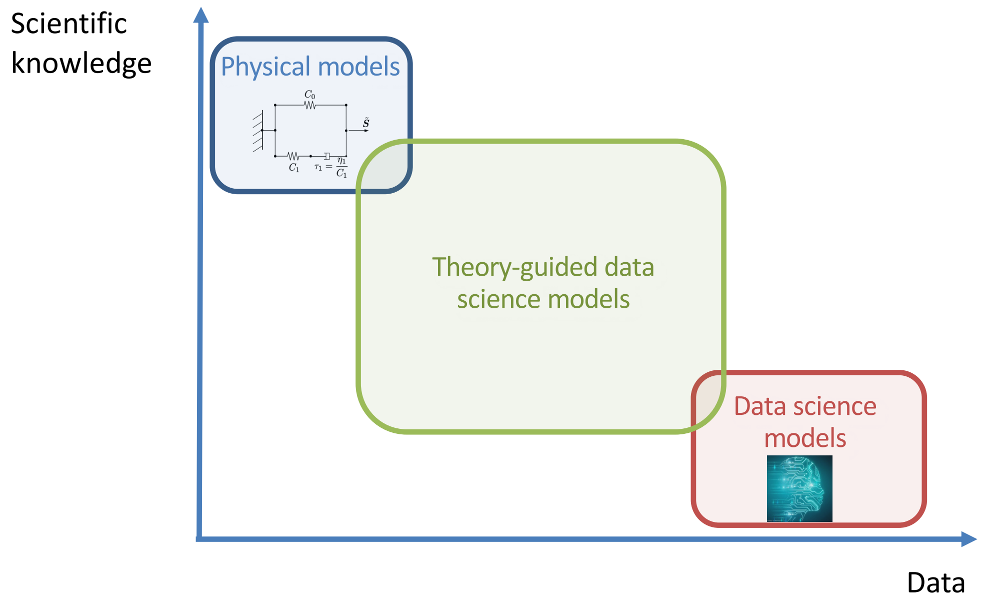
High computational costs may limit the use of the FE models.

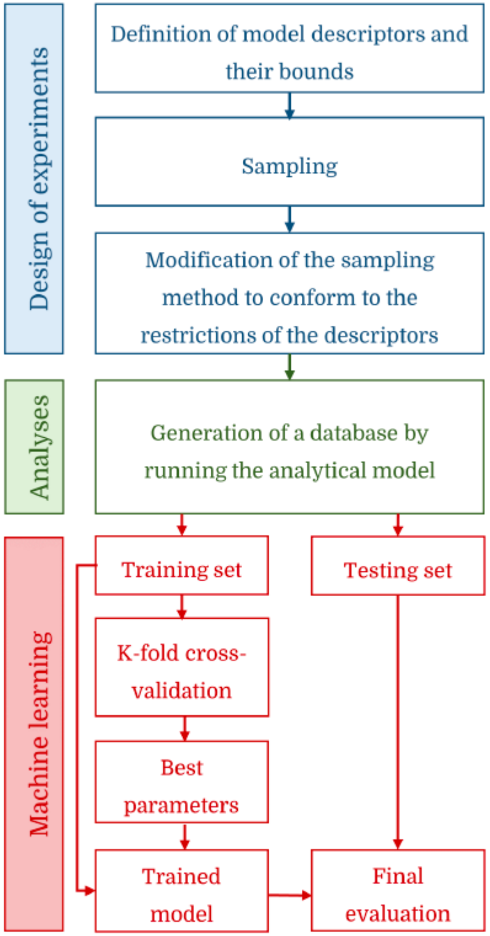
*Questions to be addressed:*

- How to define a machine learning-based approach to predict composite design allowables?
- What is the best machine learning algorithm to predict composite design allowables?

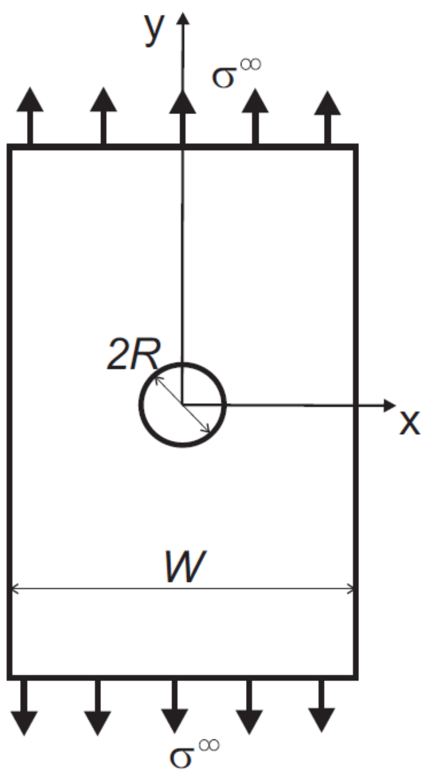
*Proposed approach:*

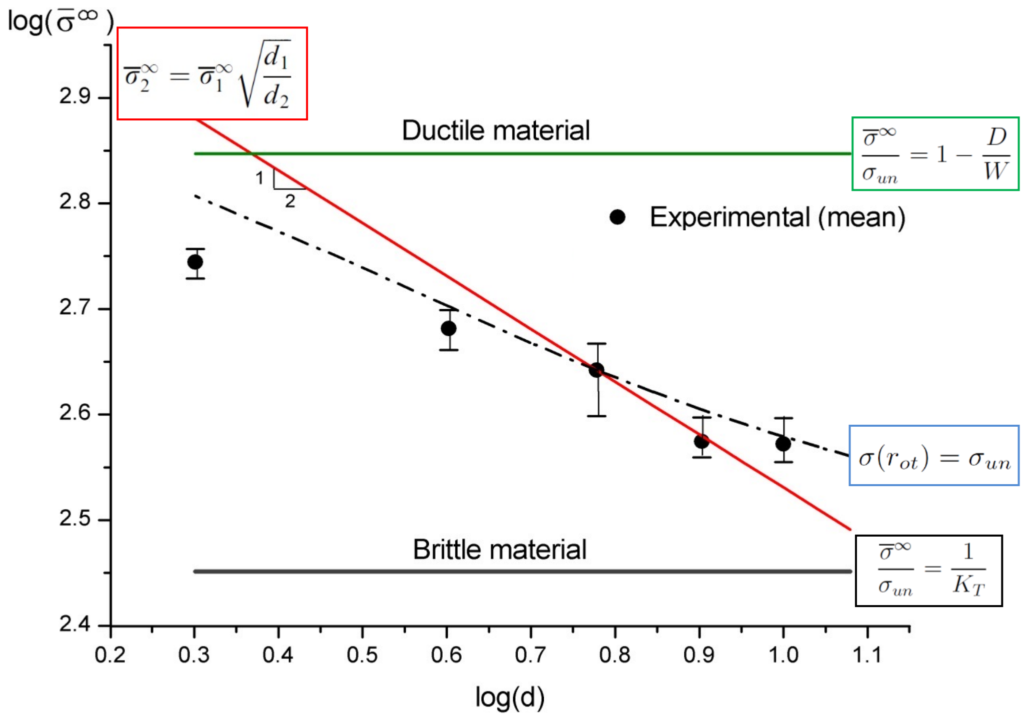
Theory guided machine learning for the generation of surrogate models:



# Analytical framework to predict the notched strength of composite laminates



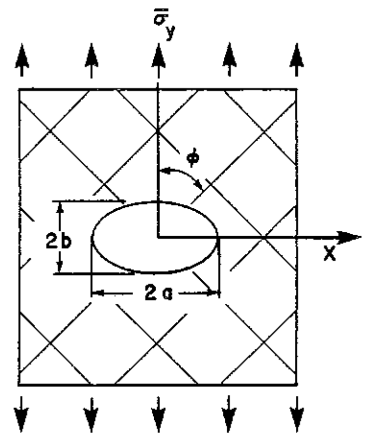

IM7-8552 CFRP, $[90/0/\pm45]_s$



(Camanho et al., **Composites Part A**, 2012).

# Analytical framework to predict the notched strength of composite laminates

$$\begin{cases} \frac{1}{l}\int_a^{a+l} \sigma_{yy}(x)dx = X_T^L \\ \int_a^{a+l} \mathcal{K}_I^2(a)da = \int_0^l \mathcal{K}_C^2(\Delta a)d(\Delta a) \end{cases}$$




$$\gamma = x/a$$

$$\lambda = b/a$$

$$\sigma_{yy}(x,0) = \sigma^\infty \mathcal{S}(\lambda, \gamma, K_T^\infty) \frac{K_T}{K_T^\infty}$$

Complex Variable Theory (Lekhnitskii)

Finite width correction factor

$$\mathcal{K}_I = \frac{1}{\sqrt{1-\left(\frac{2a}{w}\right)^2}} \sigma^\infty \sqrt{\pi a} \quad , \lambda \to 0$$

$$\mathcal{K}_I = \sigma^\infty F_h F_w \sqrt{\pi a} \quad , \lambda = 1$$

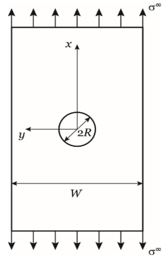
(Camanho et al., **Composites Part A**, 2012).

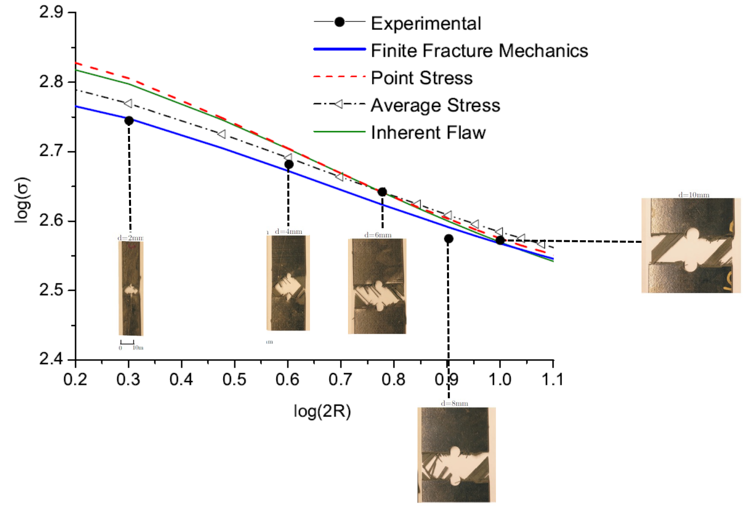
New development required for the implementation of the proposed approach: stress intensity factor for non-quasi-isotropic laminates.

(Catalanotti, Salgado, Camanho, **Engineering Fracture Mechanics**, 2021).

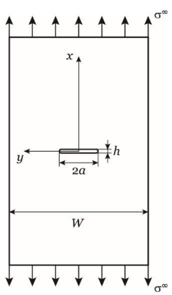
# Analytical framework to predict the notched strength of composite laminates

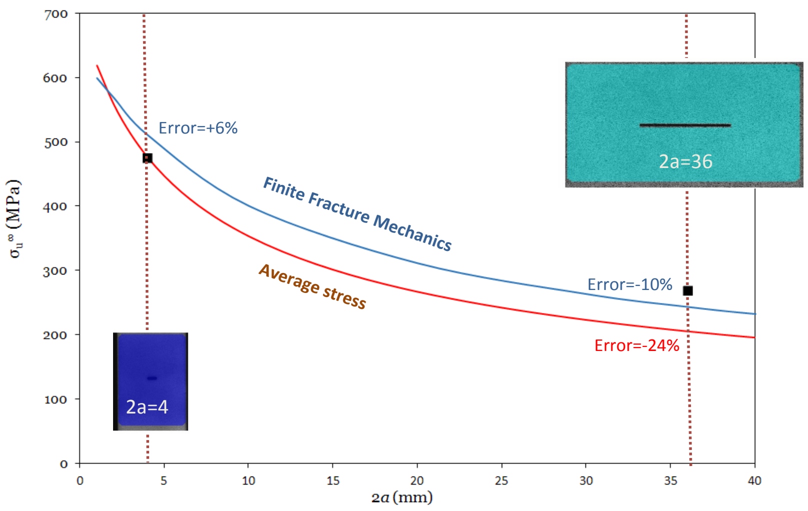
λ=1




λ→0



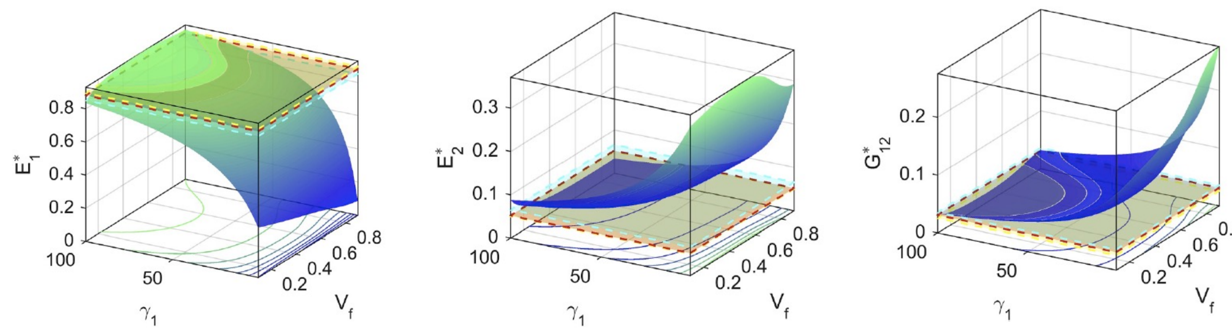
Good accuracy, but the model relies on laminate properties.

$$\begin{cases} \dfrac{1}{l}\displaystyle\int_a^{a+l} \sigma_{yy}(x)dx = X_T^L & \text{Laminate property} \\ \displaystyle\int_a^{a+l} \mathcal{K}_I^2(a)da = \int_0^l \mathcal{K}_C^2(\Delta a)d(\Delta a) & \text{Laminate property} \end{cases}$$

$A_{ij}$ (4 ply elastic properties)

## Laminate elastic properties

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & 2Q_{66} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & 2G_{12} \end{bmatrix}$$



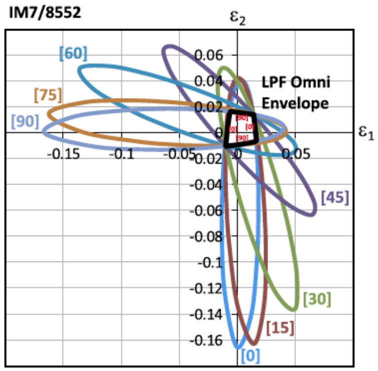

$\gamma_1 \gtrsim 50$

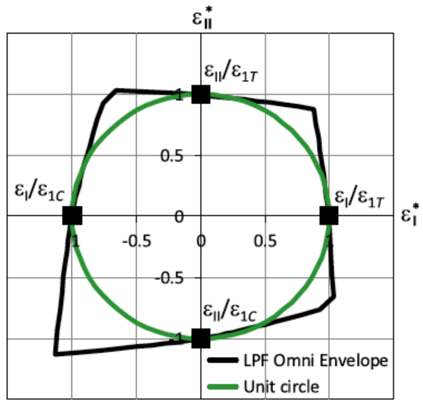
$0.5 < V_f < 0.7$

| Master Ply | $E_1^*$ | $E_2^*$ | $G_{12}^*$ |
|---|---|---|---|
| [0] | 0.880 | 0.052 | 0.031 |

$$\mathrm{Tr}(\mathbf{Q}) = \frac{E_1}{E_1^*}$$

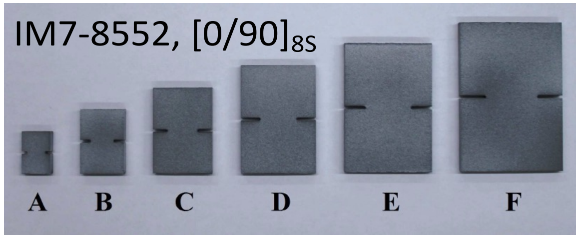
# Laminate strength



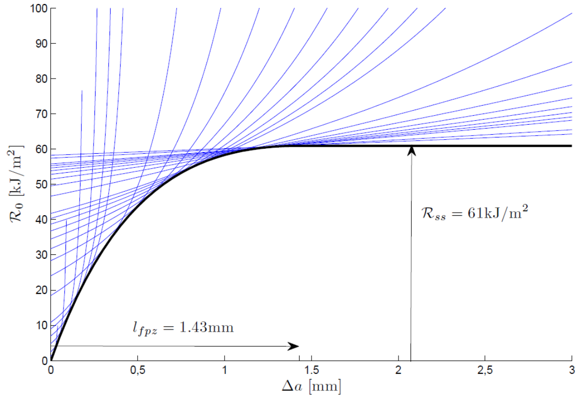



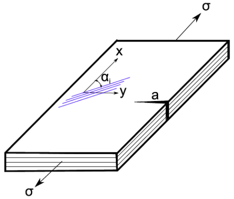

# Laminate fracture toughness

$$\mathcal{G}_{Ic}^{0}$$




(Catalanotti, Xavier, Camanho, **Composites – A**, 56, 2014).




$$\Omega_0^{(i)} = \frac{\bar{\sigma}^{(i)}}{\bar{\sigma}^0}$$

$$\bar{\sigma}^{(i)} = \frac{\mathcal{K}_{Ic}^{(i)}}{\chi^{(i)} Y \sqrt{\pi a}}$$

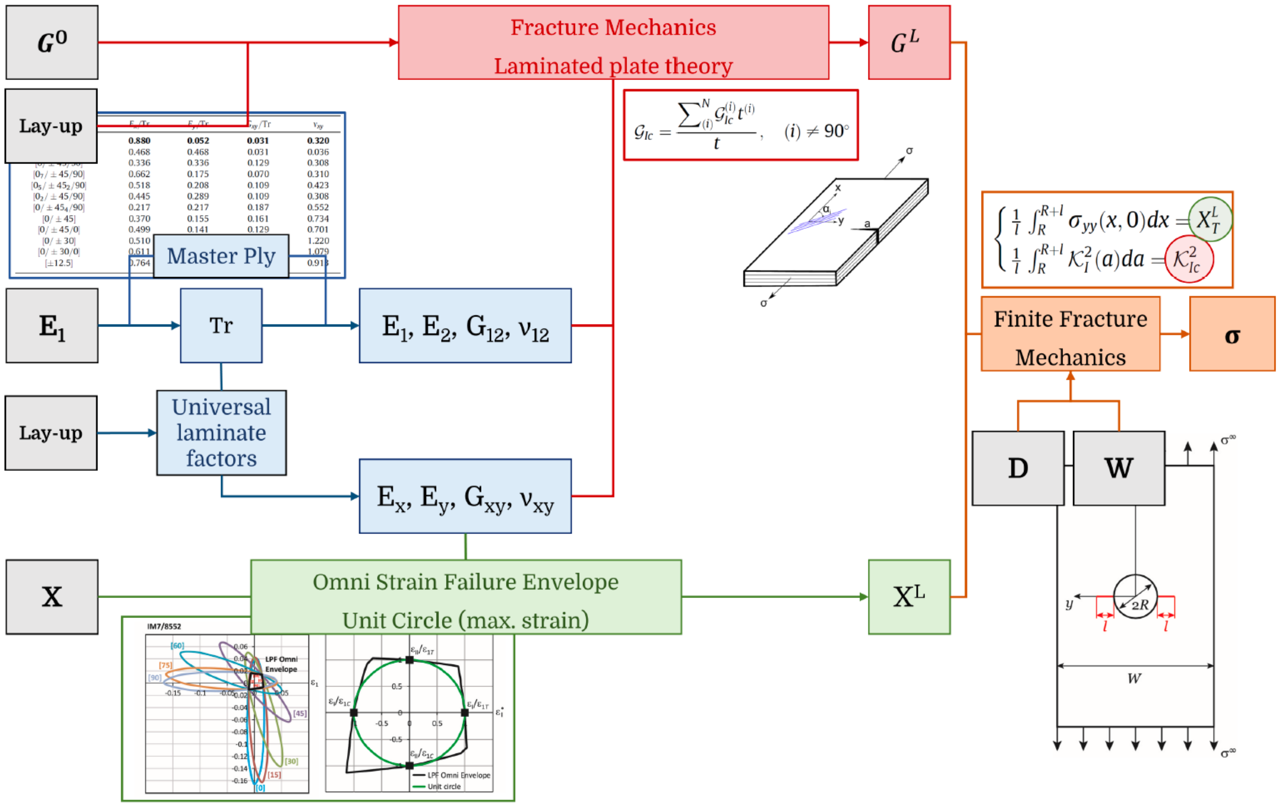
$$\bar{\sigma}^{0} = \frac{\mathcal{K}_{Ic}^{0}}{\chi^{0} Y \sqrt{\pi a}}$$

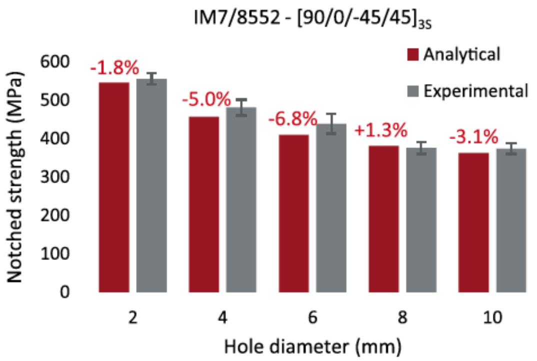
$$\mathcal{K}_{Ic}^{(i)} = \frac{\chi^{(i)}\Omega_0^{(i)}}{\chi^0}\mathcal{K}_{Ic}^{0} \Rightarrow \mathcal{G}_{Ic}^{(i)} = \frac{\left[\mathcal{K}_{Ic}^{(i)}\right]^2}{E_{eq}^{(i)}}$$

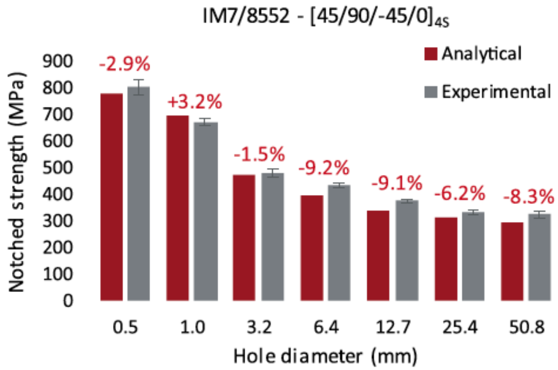
$$\mathcal{G}_{Ic}^{L} = \frac{\sum_{(i)}^{N} \mathcal{G}_{Ic}^{(i)} t^{(i)}}{t^L}, \ (i) \neq 90^\circ$$

(Camanho, Catalanotti, **Engineering Fracture Mechanics**, 78, 2011).

# Analytical framework to predict the notched strength of composite laminates



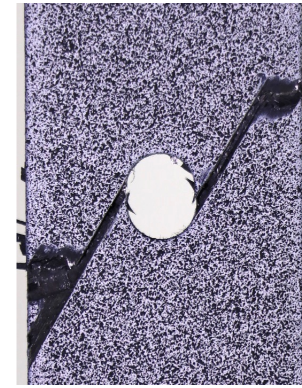

(Furtado, Arteiro, Camanho et al., **Composites Part A**, 2017).

T700/M21 75gsm
$[90/\pm45/90/\pm45/90/0]_{4s}$

15° off-axis

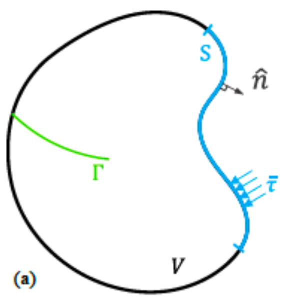
45° off-axis

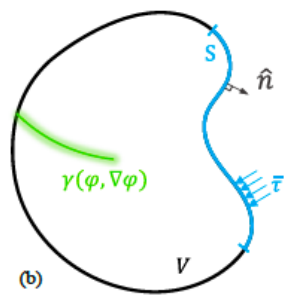
60° off-axis



Phase-field methods for fracture: variational form of the Griffith thermodynamic balance

Sharp crack




Approximated diffuse crack band – phase field

$$\Pi(\mathbf{u}) = \int_\Omega \psi_0(\mathbf{u})\mathrm{d}\Omega + \int_\Gamma G_c \mathrm{d}S + W_{ext}(\mathbf{u}) \qquad \int_\Gamma G_c \mathrm{d}S \approx \int_\Omega G_c \gamma(\phi;\nabla\phi)\mathrm{d}\Omega$$

$$\Pi(\mathbf{u}) \approx \Pi(\mathbf{u},\phi) = \int_\Omega \psi(\mathbf{u},\phi)\mathrm{d}\Omega + \int_\Omega G_c \gamma(\phi;\nabla\phi)\mathrm{d}\Omega + W_{ext}(\mathbf{u})$$

# Analytical framework to predict the notched strength of composite laminates

Modeling a thin-ply laminate at the macroscale, in an **equivalent single layer form**, requires proper consideration of:

- Elastic Anisotropy
- **Fracture energy anisotropy**

$$G_c(\theta) = g_c\sqrt{1 + a_1 \sin^2(\theta) + a_2 \cos^2(\theta)}$$

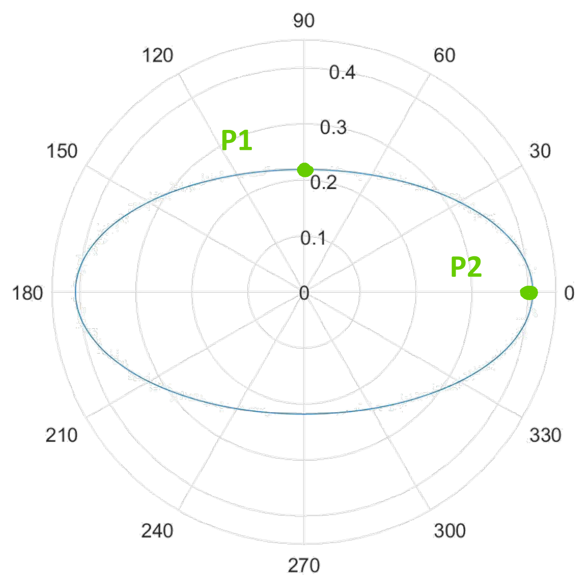
*The crack surface density function:*

*__structural tensor__ that defines preferred fracture directions*

$$\gamma(\varphi, \nabla\varphi) = \frac{1}{2l}\left(\varphi^2 + \frac{l}{2}\nabla\varphi \cdot \mathbf{A} \cdot \nabla\varphi\right)$$

$$\mathbf{A} = 1 + a_1 \boldsymbol{a}_1 \otimes \boldsymbol{a}_1 + a_2 \boldsymbol{a}_2 \otimes \boldsymbol{a}_2$$

*scaling constants*

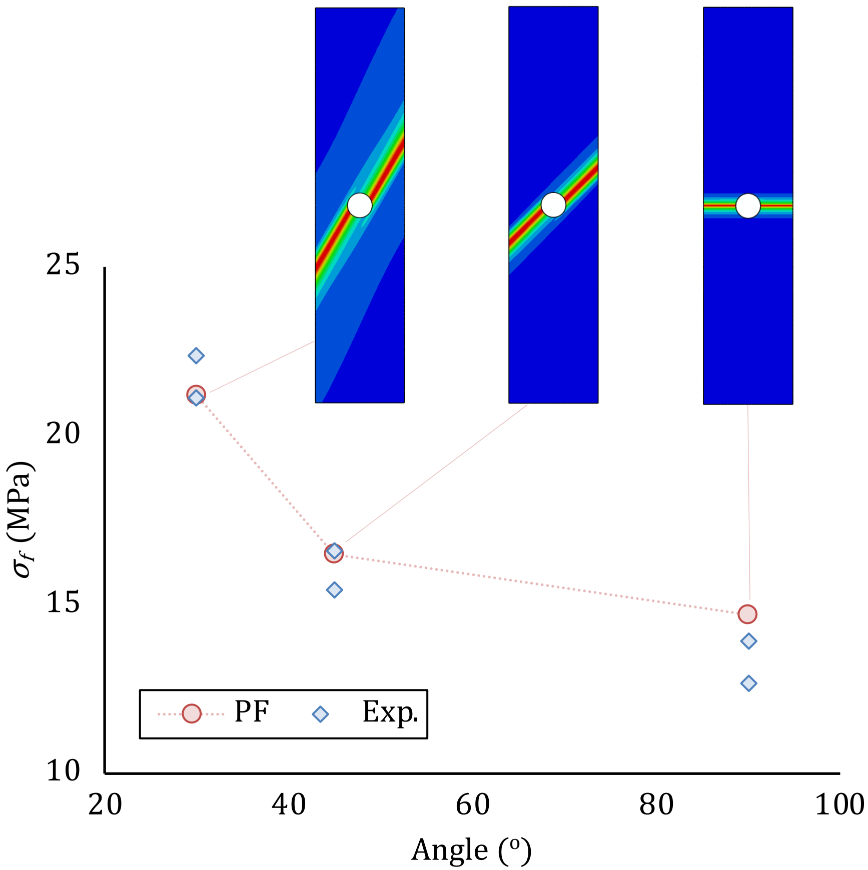



Known Points **P1, P2**

$$\boldsymbol{a_1} = \left(\frac{\boldsymbol{G_c(90°)}}{\boldsymbol{g_c}}\right)^2 - \mathbf{1} \qquad \boldsymbol{a_2} = \left(\frac{\boldsymbol{G_c(0°)}}{\boldsymbol{g_c}}\right)^2 - \mathbf{1}$$

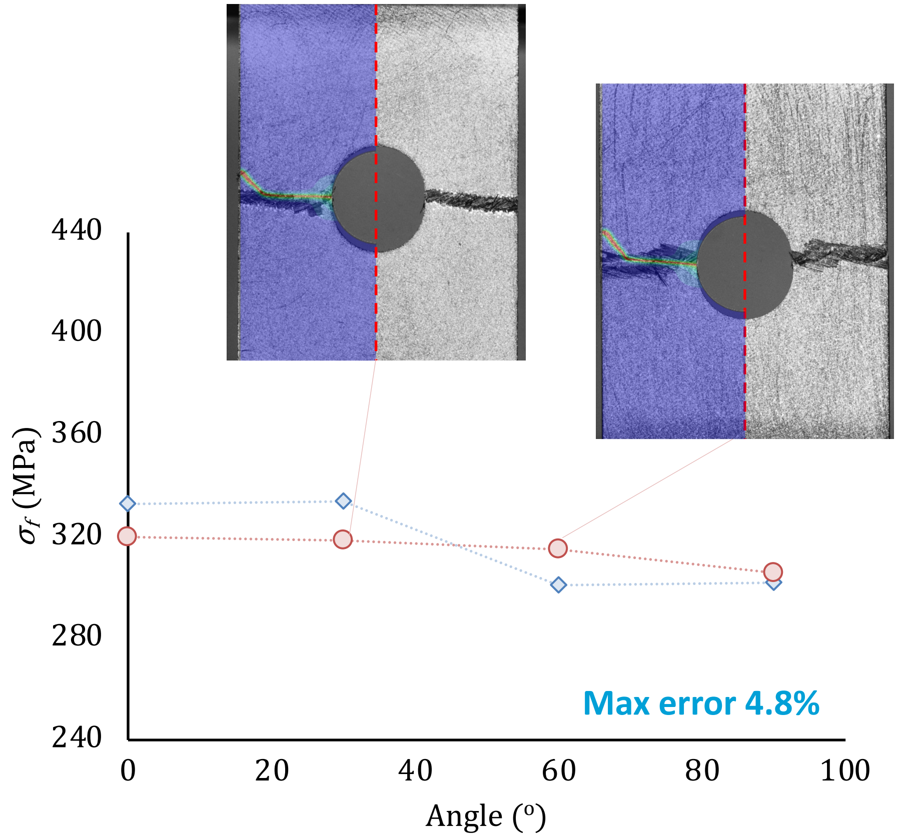
The scaling constants typically used as numerical inputs are **a material input** and represent the full fracture toughness of the laminate

UD off-axis response is seamlessly captured:




Off-axis predictions:

$[45/-45/0/45/-45/90/0/45/-45/90/0]_S$






(A Mitrou, PP Camanho, et al., International Journal of Solids and Structures, 2022).

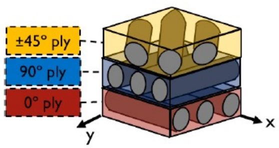
## Design of experiments and computational analysis

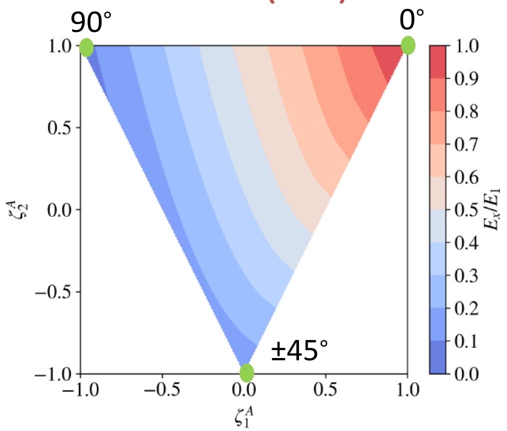
*Laminate definition*

Considering each ply orientation of a given stacking sequence as an input parameter yields a highly dimensional space that is unsuitable for machine learning.

Two classes of laminates considered: Legacy Quads (LQ) and Double-Double (DD).

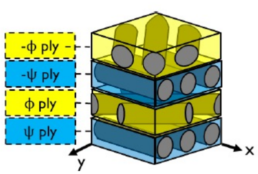
### Legacy Quads

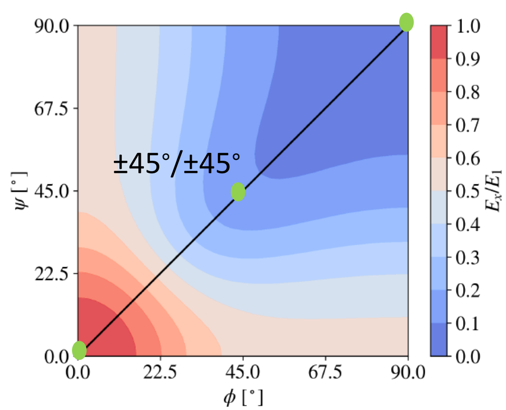



Two lamination parameters: $\zeta^A_{\{1,2,3,4\}} = \frac{1}{h}\sum_{i=1}^{N}\begin{Bmatrix}\cos(2\theta_i)\\ \cos(4\theta_i)\\ \sin(2\theta_i)\\ \sin(4\theta_i)\end{Bmatrix}(z_i - z_{i-1})$

### Double-Double




Two fibre orientation angles: $[\pm\phi/\pm\psi]_n$

*Definition of dimensionless parameters*

$$\sigma = f_1\left(E_1,\ X_T,\ \mathcal{G}_0,\ D,\ W/D,\ \text{lay-up}\right) \quad \text{lay-up} = f_2\left(\alpha,\ \beta\right)$$

Legacy Quads: $\alpha = \zeta_1^A$, $\beta = \zeta_2^A$

Double-Double: $\alpha = \phi$, $\beta = \psi$

Reduction of input parameters - Buckingham's $\Pi$ theorem:

$$\frac{\sigma}{X_T} = f\left(\frac{\mathcal{G}_0 E_1}{D X_T^2},\ W/D,\ \alpha,\ \beta\right)$$

*Design of experiments*

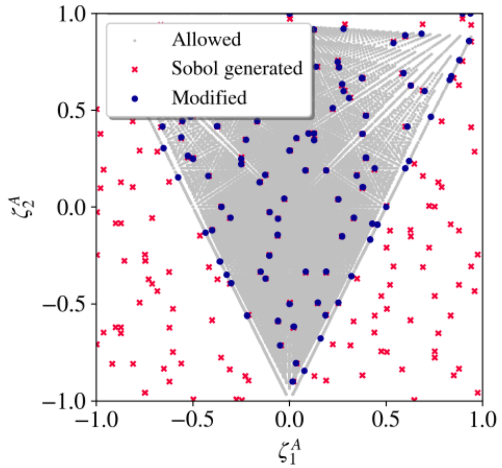
Sobol sequence using SALib Python library; material, geometric and laminate bounds:

| | Legacy Quad | Double-Double |
|---|---|---|
| Material Property | $E_1 \in [150, 200]$ GPa<br>$X_T \in [2000, 2500]$ MPa<br>$\mathcal{G}_0 \in [150, 250]$ N/mm | $E_1 \in [150, 200]$ GPa<br>$X_T \in [2000, 2500]$ MPa<br>$\mathcal{G}_0 \in [150, 250]$ N/mm |
| Geometric | $D \in [1, 12]$ mm<br>$W/D \in [3, 8]$ | $D \in [1, 12]$ mm<br>$W/D \in [3, 8]$ |
| Layup | $\zeta_1^A \in [-1, 1]$<br>$\zeta_2^A \in [-1, 1]$ | $\phi \in [0, 90]^\circ$<br>$\psi \in [0, 90]^\circ$ |

## Legacy Quads

Requires the solution of an inverse problem: from the lamination parameters to the lay-up.

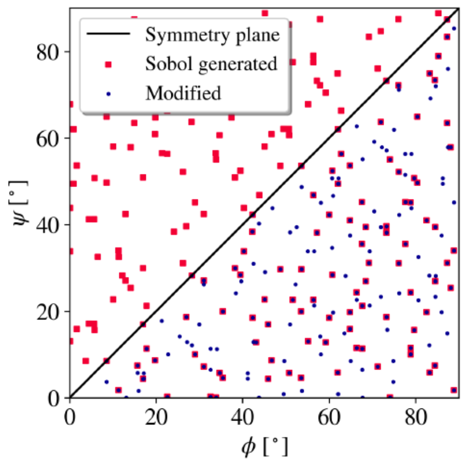
A database relating all possible $\zeta_1^A/\zeta_2^A$ combinations to the corresponding lay-up was created with the following restrictions: up to 32 plies and balanced laminates.




Input data point: $x_i = [E_1,\ X_T,\ \mathcal{G}_0,\ D,\ W/D,\ \zeta_1^A,\ \zeta_2^A]_i$

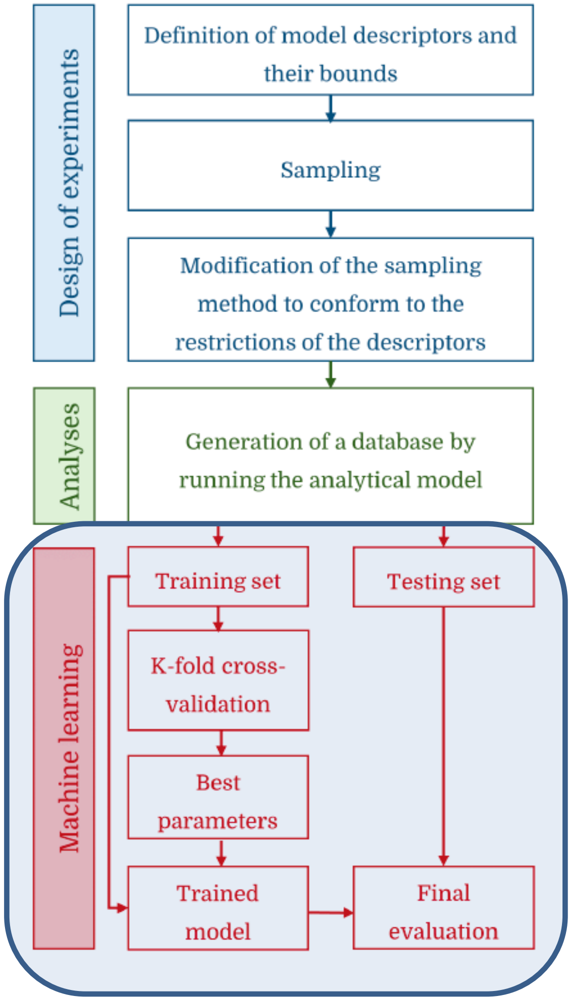
1. The laminate parameters ( $\zeta_1^A$ and $\zeta_2^A$ ) are converted into a lay-up by accessing the database.
2. Computation of the notched strength $\sigma$ and normalized $\sigma/X_T$ notched strength.
3. Conversion of the 7D space into a 4D space: $x_i^* = [\frac{\mathcal{G}_0 E_1}{D X_T^2},\ W/D,\ \zeta_1^A,\ \zeta_2^A]_i$

## Double-Double




Input data point: $x_i = [E_1,\ X_T,\ \mathcal{G}_0,\ D,\ W/D,\ \phi,\ \psi]_i$

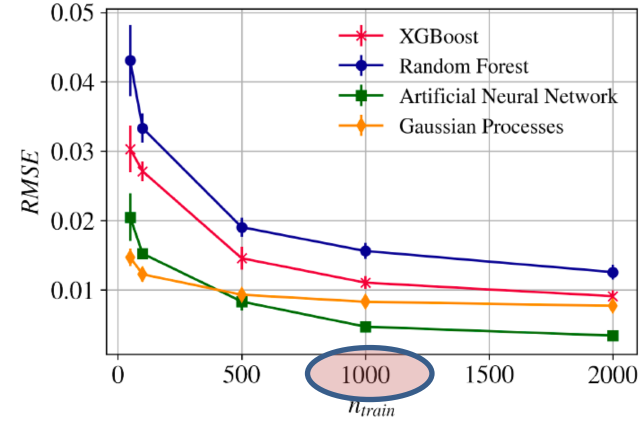
1. The laminate parameters ( $\phi$ and $\psi$ ) are converted into a lay-up $[\pm\phi/\pm\psi]$ .
2. Computation of the notched strength $\sigma$ and normalized $\sigma/X_T$ notched strength.
3. Conversion of the 7D space into a 4D space: $x_i^* = [\frac{\mathcal{G}_0 E_1}{D X_T^2},\ W/D,\ \phi,\ \psi]_i$



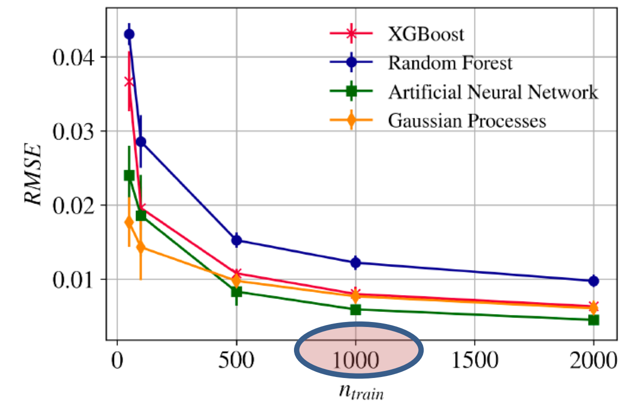

## Algorithms used:

- Gaussian Processes.
- Artificial Neural Networks
- Random Forests.
- XGBoost.

## Legacy Quads




## Double-Double

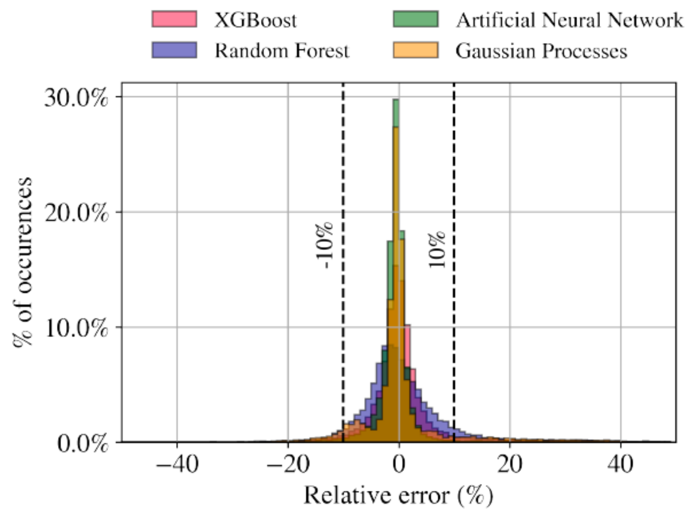


# Generation of B-basis allowables using machine learning

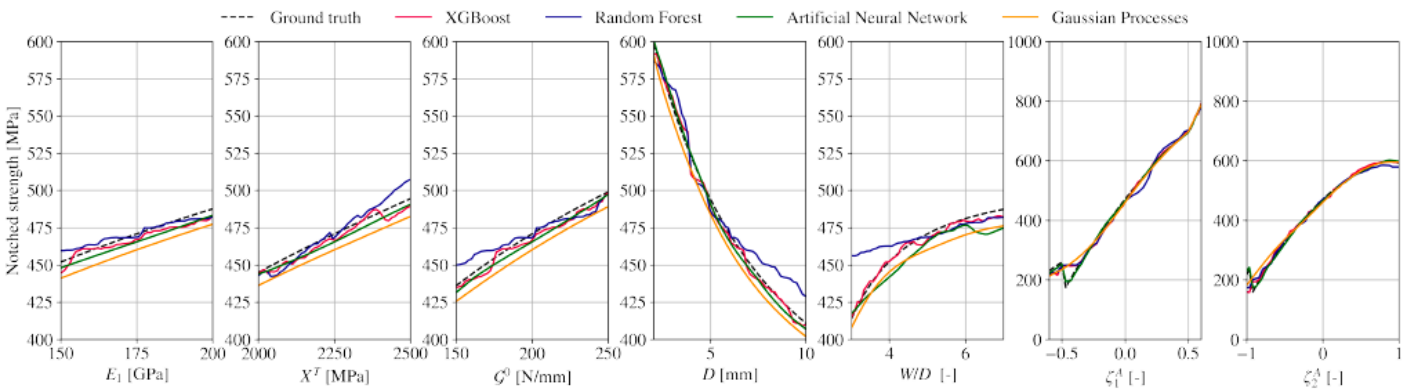
Representation of the design space – 1,000 training points and 9,000 testing points

## Legacy Quads

Relative error



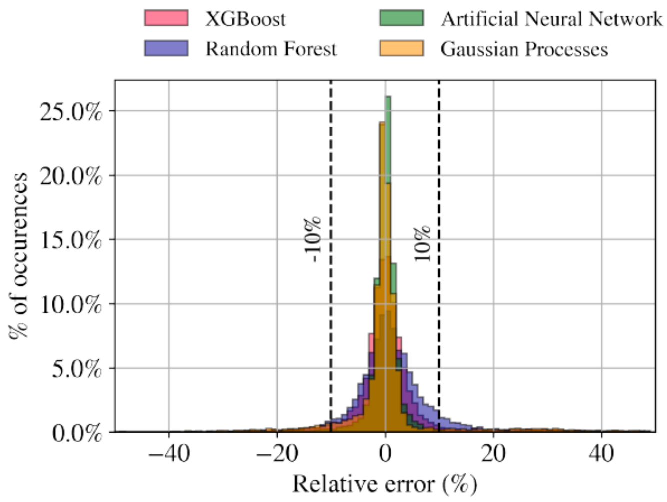

Sensitivity analysis



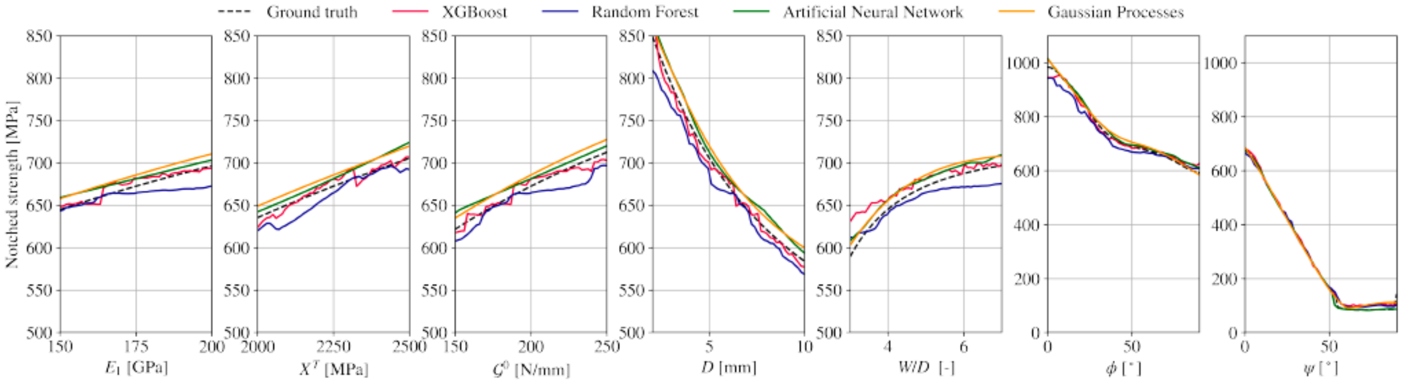
# Generation of B-basis allowables using machine learning

## Double-Double

Relative error




Sensitivity analysis



## Generation of B-basis allowables using machine learning

B-basis – 95% lower confidence bound on the tenth percentile of a specified population; it accounts for geometric and material variabilities.

Monte Carlo-based approach used:

1. 10,000 material $(E_1,\ X^L,\ \mathcal{G}_0)$ and geometric $(D \text{ and } W)$ parameters are generated following a given statistical distribution.

2. The distribution of the notched strength is calculated using the analytical model and the trained machine learning algorithms.

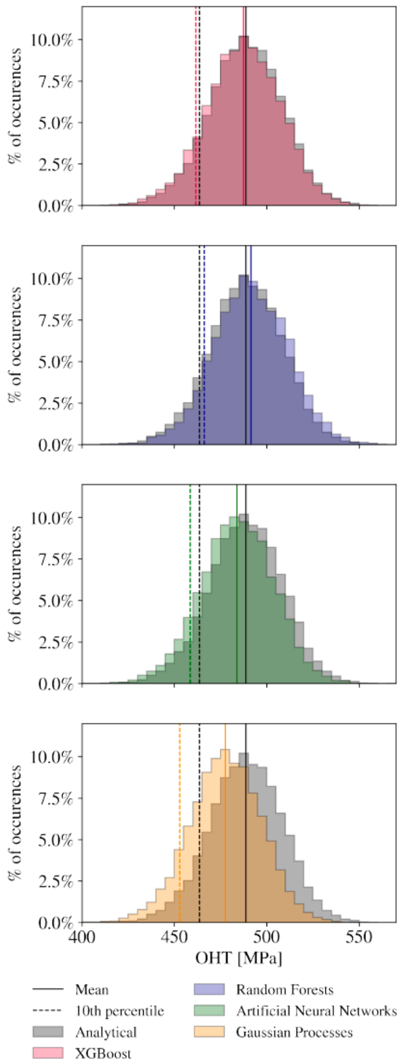
3. The $10^{th}$ percentile of the distribution on the notched strength is obtained and taken as the B-basis value.

IM7/8552, 3 < W/D < 8, D=2, 4, 6, 8 and 10mm

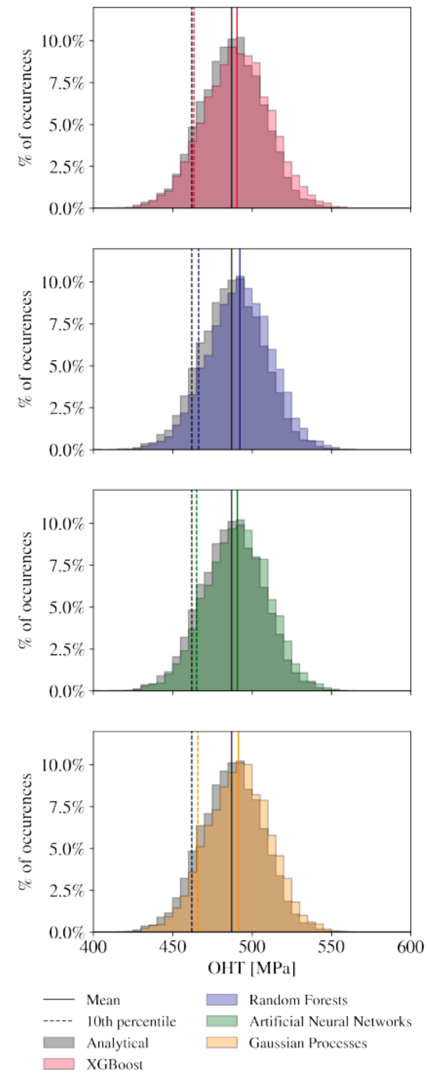
Legacy Quad: $(90/0/-45/45)_{3s}$ Double-Double: (45/-45/20/-20)

Distribution of the notched strength, B-basis and mean value of the notched strength. D=6mm, W=36mm

Legacy Quad

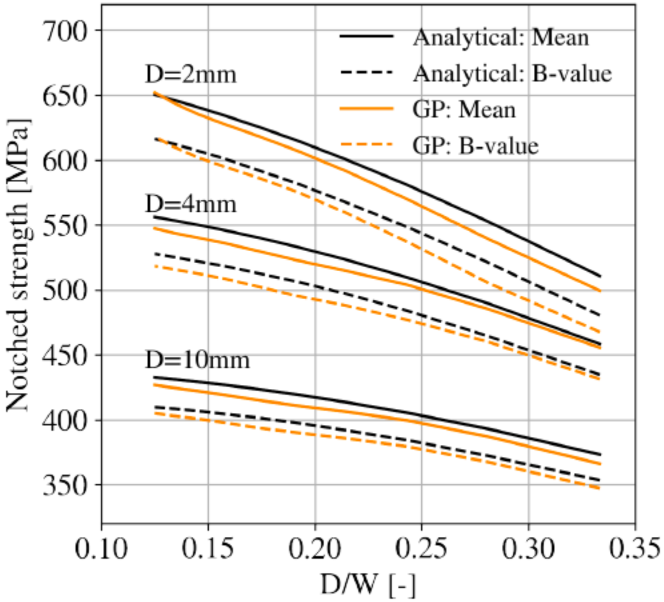



Double-Double

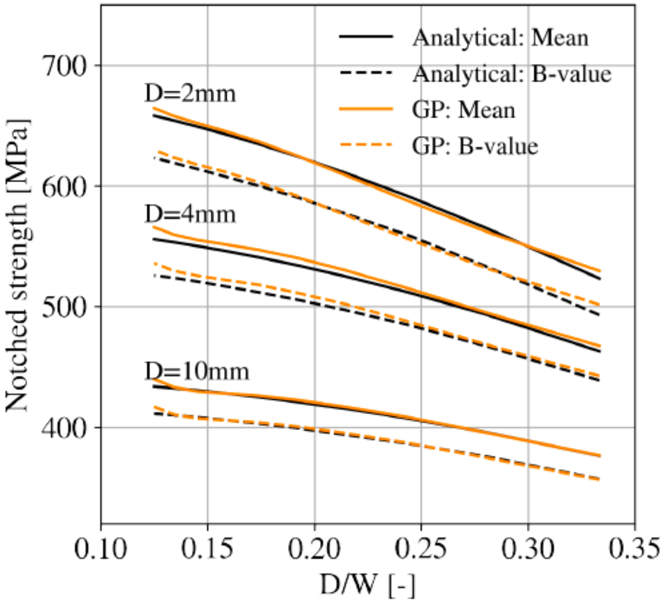


# Design charts

## Legacy Quad




## Double-Double



# Conclusions

- ✓ Using analysis models and restrictions in the definition of lay-ups it is possible to define machine learning models to predict the notched strength of composite laminates.

- ✓ Gaussian Processes achieved the best performance for a small number of data points.

- ✓ Artificial Neural Networks outperformed all other algorithms for larger data sets.

- ✓ All machine-learning models provided good estimations of B-basis design allowables.

- ✓ Gaussian processes is the preferred machine learning model taking into account:

  - ✓ Continuous and accurate representation of design space.
  - ✓ Low errors of the predictions.
  - ✓ Fast learning process.
  - ✓ Low number of hyperparameters to optimize.
  - ✓ Better performance for small training sets.

<u>More information</u>: Carolina Furtado et al., *A methodology to generate design allowables of composite laminates using machine learning*, **International Journal of Solids and Structures**.

# Acknowledgements

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Funded by the European Commission under the Marie Skłodowska-Curie Actions
Grant Agreement n° 861061
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